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Small Scale Structure Formation in Chameleon Cosmology

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Abstract

Chameleon fields are scalar fields whose mass depends on the ambient matter density. We investigate the effects of these fields on the growth of density perturbations on sub-galactic scales and the formation of the first dark matter halos. Density perturbations on comoving scales $R < 1\text{pc}$ go non-linear and collapse to form structure much earlier than in standard ΛCDM cosmology. The resulting mini-halos are hence more dense and resilient to disruption. We therefore expect (provided that the density perturbations on these scales have not been erased by damping processes) that the dark matter distribution on small scales would be more clumpy in chameleon cosmology than in the ΛCDM model.

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1 Introduction

Cosmological observations [1, 2] indicate that more than two thirds of the energy density of the Universe is in a component with negative pressure. Candidates for this missing energy, which is causing the Universe to accelerate, include a cosmological constant and scalar field models with equation of state $w \neq 1$, often referred to as quintessence [3]. In these models, the scalar field is either decoupled from Cold Dark Matter (CDM) or couples to CDM but not to the baryons (coupled quintessence)

In order to generate the present day acceleration the scalar field in these models must be evolving slowly and hence have a tiny mass, of order the present day Hubble constant, $H_0 \sim 10^{-33}\text{eV}$. Since the mass of a quintessence field is very small, it can give rise to a new long-range force. Such a force has not been observed and consequently the coupling of the quintessence field to matter must be very small. Unfortunately, in effective theories derived from string theory nearly massless fields couple to matter with gravitational strength and would produce unacceptably large violations of the equivalence principle. Khoury and Weltman have proposed a scenario where a scalar field with gravitational strength coupling to matter can evolve on a Hubble timescale and generate the present day acceleration while evading all existing tests of gravity [4]. The key feature of this scenario is that the scalar field, which is dubbed the chameleon, has a mass which depends on the local background matter density. On Earth where the density is high, the Compton wavelength of the field is sufficiently small to satisfy all current tests of gravity, while on cosmological scales, where the density is tiny, the field has a much smaller mass and can drive the present day acceleration [5]. The field, however, is heavier than in standard quintessence models ($m_{\text{cham}} \gg H$). In the solar system, where the density is many orders of magnitude smaller than on Earth, the chameleon is essentially a free field and mediates a long range force which could be detected by upcoming satellite experiments [6].

The cosmological history of the chameleon field was studied in Ref. [5]. It was shown that there is an attractor solution, analogous to the tracker solution in quintessence models, where the chameleon quickly settles into the minimum of its effective potential, and for a broad class of potentials and initial conditions the chameleon can satisfy all observational constraints. The evolution of the density perturbations was also investigated. It was shown that perturbations on scales smaller than the scale of the chameleon feel a larger effective Newton's constant which causes them to grow more rapidly. The length scale of the chameleon ($\mathcal{O}(100\text{pc})$ at present) is somewhat smaller than the scales probed by large scale structure observations. There has, however, recently been much interest in the properties of the first generation of dark matter structures to form in the Universe, and the possibility that they may leave an observable imprint in the present day dark matter distribution [7, 8, 9, 10, 11].

In this paper we examine the effects of the chameleon on the density perturbations on sub-galactic scales and the properties (formation epoch and over-density) of the first gravitationally bound structures to form. In section 2 we review the necessary aspects of the chameleon and its dynamics from Ref. [5]. In section 3 we extend the calculations of the evolution of density perturbations in Refs. [9, 11] to include the effects of the chameleon.

Finally in section 4 we examine the effect of the chameleon on the formation of small scale structure.

2 The chameleon and its dynamics

The action describing the chameleon field, χ , matter and gravity has the general form

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \frac{1}{2} (\partial\chi)^2 - V(\chi) \right] - \int d^4x \mathcal{L}_{\text{m}}(\chi_{\text{m}}^{(i)} g_{\mu\nu}^{(i)}), \quad (1)$$

where $M_{\text{Pl}} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass and $\chi_{\text{m}}^{(i)}$ are the various matter fields. The metrics governing the excitations of the matter fields are related to the Einstein frame metric $g_{\mu\nu}$ via the conformal rescaling $g_{\mu\nu}^{(i)} = \exp(2\beta_i \phi/M_{\text{Pl}}) g_{\mu\nu}$ where the β_i are dimensionless quantities of order unity [12]. Notice that the scalar field couples to all matter species including the baryons. As an example, this is the case for the radion field describing the interbrane distance in brane world models based on the Randall–Sundrum model [13] in which branes are nearby, for which $\beta_i \equiv \beta = 1/\sqrt{6}$.

By varying the action with respect to χ it can be shown [12, 14, 4, 5] that the dynamics of χ are governed by the effective potential

$$V_{\text{eff}}(\chi) = V(\chi) + \Sigma_i \rho_i \exp(\beta_i \chi/M_{\text{Pl}}), \quad (2)$$

where the matter density ρ_i is defined as $\rho_i \equiv -g_{(i)}^{\mu\nu} T_{\mu\nu}^{(i)} \exp(3\beta_i \chi/M_{\text{Pl}})$ so that it is independent of χ and conserved in the Einstein frame. If $V(\chi)$ decreases monotonically with increasing χ and $\beta_i > 0$, this potential has a minimum, χ_{min} , which increases with decreasing ρ_i . The mass of small fluctuations about χ_{min} increases with increasing χ_{min} so that the chameleon can evade local tests of the equivalence principle and fifth forces, due to the high local density.

Fiducial potentials of the form

$$V(\chi) = M^4 \exp(M^n/\chi^n), \quad (3)$$

with $M = 10^{-3}\text{eV}$ so as to produce the observed present day dark energy density (and also satisfy local tests of general relativity) were studied in Ref. [5]. Assuming for simplicity a single matter component with density ρ_{m} and coupling β , the field value at the minimum of the effective potential satisfies

$$\left(\frac{M}{\chi_{\text{min}}(t)} \right)^{n+1} = \frac{\beta}{n} \frac{M}{M_{\text{Pl}}} \frac{\rho_{\text{m}} \exp(\beta \chi_{\text{min}}(t)/M_{\text{Pl}})}{V(\chi_{\text{min}}(t))}. \quad (4)$$

It was found that for a wide range of initial conditions the chameleon field reaches the attractor solution with $\chi(t) = \chi_{\text{min}}(t)$ before big bang nucleosynthesis and has a cosmological evolution in accordance with all observational constraints.

The presence of a chameleon field can effect the growth of structure in particular on small scales. In the following, we will consider the modifications to the growth factor due to the gravitational effects induced by the chameleon.

3 Perturbation evolution

We will follow Refs. [9, 11] and work in the longitudinal gauge, we will however use the notation of Ma and Bertschinger [15]. The perturbed line element reads

$$ds^2 = a^2(\eta) \left[- (1 + 2\Psi) d\eta^2 + (1 - 2\phi) g_{ij} dx^i dx^j \right]. \quad (5)$$

The equations of motion for the CDM density contrast δ_c and the divergence of the CDM velocity field Θ_c can be obtained from the energy-momentum conservation equation, which contains an additional term due to the exchange of energy with the chameleon

$$T_{(i)}^{\mu\nu}{}_{;\mu} = \beta_{(i)} (\partial^\nu \chi) T_{(i)}. \quad (6)$$

Here, i stands for the component i and $T = T^\mu{}_\mu$. The equations of motion are then given by (the dot represents the derivative with respect to η and $\mathcal{H} \equiv (da/d\eta)/a$)

$$\dot{\delta}_c = -\Theta_c + 3\dot{\phi} + \beta(\delta\chi), \quad (7)$$

$$\dot{\Theta}_c = -(\mathcal{H} + \beta\dot{\chi})\Theta_c + k^2(\Psi + \beta\delta\chi). \quad (8)$$

The perturbed Klein-Gordon equation for the chameleon field χ is given by

$$(\delta\chi)'' + 2\mathcal{H}(\delta\chi)' + \left(k^2 + a^2 \frac{\partial^2 V}{\partial \chi^2} \right) \delta\chi + 2\Psi \left(\frac{\partial V}{\partial \chi} + \beta\rho_c \right) a^2 - 4\dot{\Psi}\dot{\chi} = -\beta\rho_c \delta_c a^2, \quad (9)$$

and we will also need one of the components of the first-order perturbed Einstein equation (Poisson's equation in the sub-horizon limit)

$$k^2\phi + 3\mathcal{H}(\dot{\phi} + \mathcal{H}\psi) = 4\pi G a^2 \delta T_0^0. \quad (10)$$

From very early times onwards (before nucleosynthesis), the mass of the chameleon field is much greater than the Hubble expansion and the field sits in the minimum of the effective potential. Consequently the interaction scale of the chameleon field is always much smaller than the horizon H^{-1} , and the evolution of perturbations on super-horizon scales is unaffected by the chameleon. Furthermore the chameleon does not couple to radiation, since it is a traceless fluid, and the evolution of perturbations deep within the radiation dominated epoch is also as in standard cosmology [15, 11]. Once $\delta_c \rho_c \gg \delta_r \rho_r$ (which for sub-galactic scales happens prior to matter-radiation equality), however, the dark matter terms dominate as the source in the Poisson equation, eq. (10), and the coupling of the chameleon to the matter density is now important. In particular, perturbations in matter will influence perturbations in the chameleon field and vice versa.

On the sub-horizon scales we are interested in we can neglect the oscillations in the perturbations in the chameleon field, and $\dot{\chi}$ is also small as the field evolves along the minimum of the effective potential [5]. Following Ref. [11] we also neglect anisotropic stress, so that $\phi = \psi$, and baryon anisotropies (but not the baryon density). The latter assumption is valid on small scales ($k > k_b \sim 10^{-3} \text{pc}^{-1}$) for $z > z_b \sim 150$ as prior

to this residual electrons allow transfer of energy between the photon and baryon fluids and thermal pressure prevents the baryon perturbations from growing [16]. Following Ref. [5, 11], eqs. (7-10) can be combined to give the following equation for the evolution of the cold dark matter density contrast:

$$\ddot{\delta}_c = -\mathcal{H}\dot{\delta}_c + \frac{3}{2} \frac{\rho_c}{\rho_c + \rho_\gamma} \left[1 + \frac{2\beta^2}{1 + \frac{a^2 V''}{k^2}} \right] \delta_c, \quad (11)$$

with $V'' = \partial^2 V / \partial \chi^2$. The effects of the chameleon manifest themselves in the second term in the square brackets. The chameleon field operates on length scales smaller than

$$\lambda_{\text{cham}}(t) \equiv \frac{1}{\sqrt{V''}}, \quad (12)$$

or equivalently for comoving wavenumbers larger than

$$k_{\text{cham}}(t) = \frac{a}{\lambda_{\text{cham}}(t)}. \quad (13)$$

For $k \ll k_{\text{cham}}(t)$ the terms in the square bracket in eq. (11) above are well approximated by 1, and the CDM density contrast evolves as in standard cosmology. For $k \gg k_{\text{cham}}(t)$, they are well approximated by $(1 + 2\beta^2)$ i.e. on these scales the growth of perturbations is governed by an effective gravitational constant given by $G(1 + 2\beta^2)$.

For $k \gg k_{\text{cham}}$, Eq. (11) can be re-written in terms of $y = a/a_{\text{eq}}$ to give

$$y(y+1)\delta_c'' + \left(1 + \frac{3}{2}y\right)\delta_c' = \frac{3}{2}(1 + 2\beta^2)(1 - f_b)\delta_c, \quad (14)$$

where $' = d/dy$ and we have introduced the baryon fraction $f_b \equiv \Omega_b/\Omega_m$. The solution to this equation is a superposition of Legendre functions of first and second kind of order ν :

$$\delta_c(k, y) = B_1(k)P_\nu\left(\sqrt{1+y}\right) + B_2(k)Q_\nu\left(\sqrt{1+y}\right). \quad (15)$$

This is the same as in the Λ CDM case [9, 11], but the degree ν of the Legendre functions in the standard case is given by

$$\nu_{\text{GR}} = \frac{-1 + \sqrt{1 + 24(1 - f_b)}}{2}, \quad (16)$$

whereas we have, for $k \gg k_{\text{cham}}(t)$,

$$\nu_{\text{cham}} = \frac{-1 + \sqrt{1 + 24(1 + 2\beta^2)(1 - f_b)}}{2}. \quad (17)$$

For the best fit WMAP Λ CDM model $f_b = 0.17$ [1] so that $\nu_{\text{GR}} = 1.8$, whereas for $\beta = 1$ $\nu_{\text{cham}} = 3.4$. For $z < z_b \sim 150$ the baryons follow the CDM (which is equivalent to setting

$f_b = 0$ in eqs. (14)-(17)) and $\nu_{\text{GR}} = 2$ for the standard cosmology and $\nu_{\text{cham}} = 3.8$ for the chameleon with $\beta = 1$ (as found in Ref. [5]). For $y \gg 1$, equivalently $z \ll z_{\text{eq}}$, $\delta_c(y)$ grows as $a^{\nu/2}$. The increase in the growth rate due to the chameleon is large compared with the suppression in growth due to baryons which occurs for $z_b < z < z_{\text{eq}}$. We therefore now neglect the baryons and set $f_b = 0$ in eqns. (16) and (17) above.

The full asymptotic late time solution is [9, 11]

$$\delta_c(y) = 6\zeta_0 c(\nu) y^{\nu/2} \left[\ln \left(\frac{k}{k_{\text{eq}}} \right) + b(\nu) \right]. \quad (18)$$

where ζ_0 is the superhorizon limit of the curvature perturbation on uniform density hypersurfaces and the constants $c(\nu)$ and $b(\nu)$ are found by matching the early time radiation domination ($y \ll 1$) expansion of eq. (15) to the sub-horizon limit of the general radiation domination solution [9, 11]

$$c(\nu) = \frac{\Gamma(1+2\nu)}{2^\nu \Gamma^2(1+\nu)}, \quad (19)$$

and

$$b(\nu) = \frac{1}{2} \ln \left(\frac{2^5}{3} \right) - \gamma_E - \frac{1}{2} - \frac{2}{\nu} - \frac{2\Gamma'(\nu)}{\Gamma(\nu)}. \quad (20)$$

where $\Gamma'(\nu)$ is the derivative of $\Gamma(\nu)$ with respect to ν . For $\nu_{\text{GR}} = 2$, $c = 1.5$ and $b = -1.7$, while for $\nu_{\text{cham}} = 3.8$, $c = 3.9$ and $b = -2.8$.

We now calculate the evolution of $k_{\text{cham}}(t)$ with time, and consequently the scales on which the chameleon effects the growth of the density contrast. During matter domination $V(\chi_{\text{min}}) \sim \text{const}$ and $\rho_m \exp(\beta\chi_{\text{min}}/M_{\text{Pl}}) \propto a^{-3}$ so that, using eq. (4),

$$\chi_{\text{min}}(t) = \chi_{\text{min}}(t_0) (1+z)^{-3/(n+1)}. \quad (21)$$

At present $\rho_m \exp(\beta\phi_{\text{min}}/M_{\text{Pl}}) \sim V(\phi_{\text{min}}) \sim M^4$ so that [5]

$$\chi_{\text{min}}(t_0) = \left(\frac{n}{\beta} \right)^{1/(n+1)} \left(\frac{M}{M_{\text{Pl}}} \right)^{n/(n+1)} M_{\text{Pl}}, \quad (22)$$

Using eqs. (12) and (13) the scale of the chameleon field varies as

$$\lambda_{\text{cham}}(t) = \lambda_{\text{cham}}(t_0) (1+z)^{[-3(n+2)/2(n+1)]}, \quad (23)$$

$$k_{\text{cham}}(t) = k_{\text{cham}}(t_0) (1+z)^{(n+4)/2(n+1)} = \frac{(1+z)^{(n+4)/2(n+1)}}{\lambda_{\text{cham}}(t_0)}. \quad (24)$$

where

$$\lambda_{\text{cham}}(t_0) = \frac{1}{\sqrt{n(n+1)}} \frac{1}{M} \left(\frac{n}{\beta} \right)^{(n+2)/2(n+1)} \left(\frac{M_{\text{Pl}}}{M} \right)^{(n+2)/2(n+1)}, \quad (25)$$

The characteristic wavenumber $k_{\text{cham}}(t)$ decreases with decreasing red-shift. At z_{eq}

$$k_{\text{cham}}(z_{\text{eq}}) = \frac{(1+z_{\text{eq}})^{(n+4)/2(n+1)}}{\lambda_{\text{cham}}(t_0)}, \quad (26)$$

where $(1+z_{\text{eq}}) = 24000\Omega_{\text{m}}h^2 \approx 3700$. For fiducial parameters $n = \beta = 1$ and $M = 10^{-3}\text{eV}$, $\lambda_{\text{cham}}(t_0) = 250\text{pc}$ and $k_{\text{cham}}(t_{\text{eq}}) = 120\text{pc}^{-1}$.

For scales $k < k_{\text{cham}}(z_{\text{eq}})$ the density contrast growth law is only modified once $k > k_{\text{cham}}(t)$. We denote the redshift at which this happens by z_{mod} , which is given by

$$(1+z_{\text{mod}}) \approx (\lambda_{\text{cham}}(t_0)k)^{2(n+1)/(n+4)}. \quad (27)$$

For $z \ll z_{\text{eq}}$ density perturbations grow as $\delta \propto a^{\nu/2}$, where for standard general relativity and $k \ll k_{\text{cham}}(t)$, $\nu = \nu_{\text{GR}}$ while for $k \gg k_{\text{cham}}(t)$, $\nu = \nu_{\text{cham}}$. The, scale dependent, additional growth in the linear density perturbation due to the chameleon is given by

$$\frac{\delta_{\text{cham}}(k, z)}{\delta_{\text{GR}}(k, z)} = \left(\frac{1+z_{\text{mod}}(k)}{1+z} \right)^{(\nu_{\text{cham}}-\nu_{\text{GR}})/2}, \quad (28)$$

where $z_{\text{mod}} \approx z_{\text{eq}}$ for $k > k_{\text{cham}}(t_{\text{eq}})$ and is given by eq. (27) for $k < k_{\text{cham}}(t_{\text{eq}})$.

In CDM cosmologies structure forms hierarchically (large halos form via the merger and accretion of smaller subhalos) and at least some substructure is expected to survive. There must be some cut-off in this process however; if the density perturbation spectrum extended down to infinitely small scales the contribution of density perturbations to the local energy density would diverge [7]. For weakly interacting massive particles (WIMPs) damping processes [17, 7, 8, 9, 11, 18], namely collisional damping (due to elastic interactions with radiation) and free-streaming, produce a cut-off in the (processed) power spectrum at $k_{\text{cut}} \sim 1\text{pc}^{-1}$ (i.e. fluctuations on scales $k > k_{\text{cut}}$ are erased). Ref. [11] found a range of values $k_{\text{cut}} \approx 0.4 - 4\text{pc}^{-1}$ for benchmark models spanning the range of plausible WIMP properties. Dirac like WIMPs, where elastic scattering is mediated by Z^0 exchange, have values of k_{cut} at the lower end of the range. Majorana WIMPs for which Z^0 exchange is suppressed, for instance neutralinos, have a wider range of k_{cut} values with more massive WIMPs having larger k_{cut} .

We plot $k_{\text{cham}}(z)$ and $z_{\text{mod}}(k)$ for $n = 1$ and $n = 2$ and also the range of k_{cut} values in figs. 1 and 2 respectively. For $n = 1$ $k_{\text{cham}}(z_{\text{eq}})$ is larger than the upper end of the range of k_{cut} values and the evolution of the surviving perturbations is initially unaffected. $k_{\text{cham}}(z)$ decreases sufficiently rapidly with decreasing red-shift, however, that $z_{\text{mod}}(k_{\text{cut}}) \gg 0$ for the entire range of k_{cut} values and the growth law of small (physical) scales is modified. For $n = 2$ $k_{\text{cham}}(z_{\text{eq}})$ is so large that the chameleon scale is beyond the cut-off scale even at late times and the growth of surviving perturbations is completely unaffected by the chameleon. We should emphasise, however, that WIMPs are not the only viable CDM candidate. There are a large number of other candidates [19] (including the arguably equally well motivated axion) whose microphysics have not yet been studied and which may have substantially different cut-off scales.

4 Small scale structure

The enhancement of the growth rate of the CDM density contrast on small scales due to the chameleon means that these scales will go non-linear, $\delta_c \sim \mathcal{O}(1)$, and collapse to

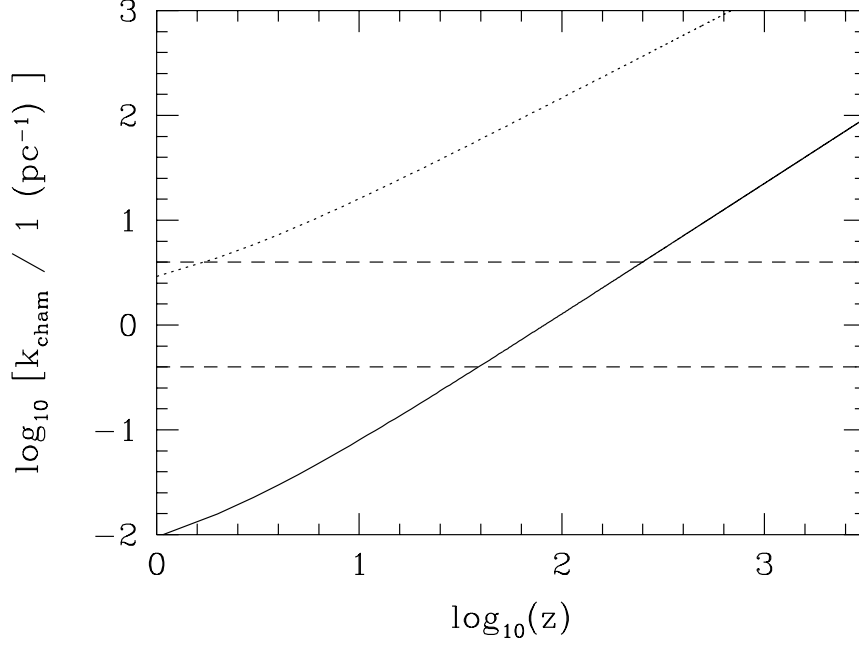


Figure 1: The red-shift dependence of the characteristic wavenumber of the chameleon, $k_{\text{cham}}(z)$, for $\beta = 1$, $M = 10^{-3}\text{eV}$ and $n = 1$ (solid line) and $n = 2$ (dotted line). The dashed lines show the range of values of k_{cut} for plausible WIMP models from Ref. [11].

form structure earlier than in standard cosmology. The red-shift at which the first typical, $1\text{-}\sigma$, fluctuations on comoving scale R go non-linear $z_{\text{nl}}(R)$ is defined via $\sigma(R, z_{\text{nl}}) = 1$, where $\sigma(R, z)$ is the mass variance calculated by integrating the density perturbation power spectrum multiplied by the Fourier transform of a window function. This calculation is not possible in this case as we only have the evolution of density perturbations on sub-galactic scales, and the change in the growth law due to the chameleon means that we can not use the value of σ_8 measured by WMAP to evade this problem, as in Ref. [9, 11]. We instead estimate the effect of the chameleon on the red-shift at which scale go non-linear by using the approximations $\delta_{\text{cham}}(k, z_{\text{nl, cham}}(k)) = 1$ and $\delta_{\text{GR}}(k, z_{\text{nl, GR}}(k)) = 1$ and using the values of $z_{\text{nl, GR}}(R)$ calculated in Ref. [11] with the approximation $k \sim 1/R$. Using eq. (28) we find

$$\frac{1 + z_{\text{nl, cham}}(k)}{1 + z_{\text{nl, GR}}(k)} = \left(\frac{1 + z_{\text{mod}}(k)}{1 + z_{\text{nl, GR}}(k)} \right)^{[1 - (\nu_{\text{gr}}/\nu_{\text{cham}})]}. \quad (29)$$

The resulting values of $z_{\text{nl, cham}}(k)$ are plotted in Fig. 4 For scales with $k < k_{\text{cham}}(t_{\text{eq}})$ we assume that ν , and hence the growth law for δ_c , changes abruptly at z_{mod} . The change would in fact occur smoothly as the chameleon term in square brackets in eq. (11) increases smoothly from 1 to $(1 + 2\beta^2)$. This approximation is reasonable however given the other uncertainties involved in the calculation.

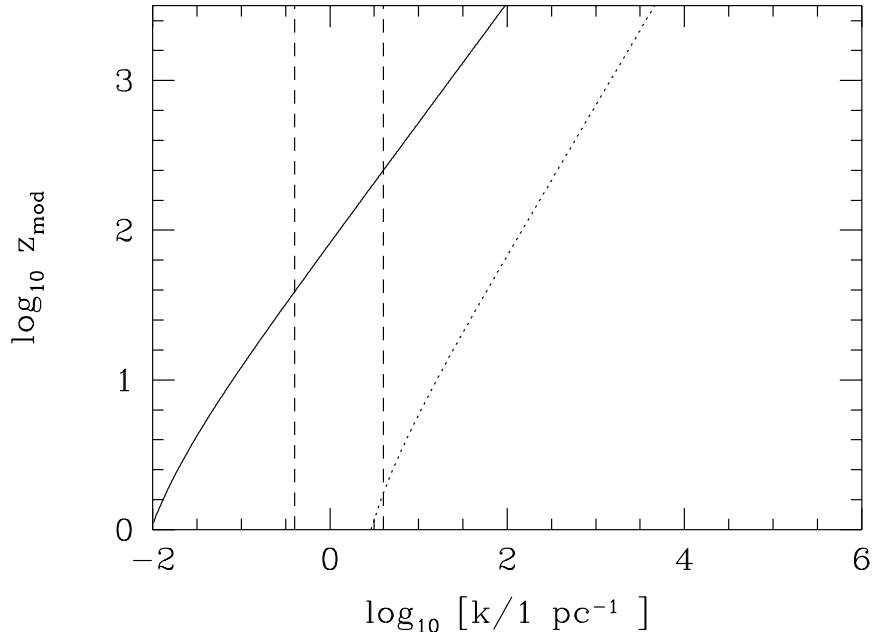


Figure 2: The red-shift at which the growth law of density is modified due to the chameleon, $z_{\text{mod}}(k)$, for $\beta = 1$, $M = 10^{-3}\text{eV}$ and $n = 1$ (solid line) and $n = 2$ (dotted line). The short-dashed lines show the plausible range of k_{cut} values for WIMPs.

A scale dependent primordial power spectrum, with spectral index $n_s > 1$, as produced by, for instance, false vacuum dominated hybrid inflation would also result in a larger than standard density contrast on small physical scales and hence earlier structure formation. We therefore also plot $z_{\text{nl,GR}}$ as calculated in Ref. [11] for a false vacuum dominated hybrid inflation model which produces a primordial power spectrum with $n_s = 1.036$, which is the maximum scale dependence allowed by the WMAP and 2dF data [20]⁵. For $n = 2$ $z_{\text{nl,cham}}(k) = z_{\text{nl,GR}}(k)$ for all $k < 10^2 \text{pc}^{-1}$. For $n = 1$, for $k > \mathcal{O}(1 \text{pc}^{-1})$ the growth law is modified sufficiently early that $z_{\text{nl,cham}}(k) \gg z_{\text{nl,GR}}(k)$. The rapid increase of $z_{\text{nl,cham}}(k)$ with increasing k is caused by the large change in the index of the growth law by the chameleon, $\delta_c \propto a^{1.9}$ compared with $\delta_c \propto a$ for standard cosmology. This resulting change in δ_c on small physical scales at late times is far larger than that produced by a primordial power spectrum with $n_s = 1.036$.

The physical properties of the first generation of WIMP halos were estimated in Ref. [11] using the spherical collapse model. The physical size of the halos post collapse is proportional to $(1 + z_{\text{nl}})^{-1}$ and hence the density contrast is proportional to $(1 + z_{\text{nl}})^3$ i.e. the

⁵A possible interaction between the chameleon and the inflaton field does not change the result for n_s , since the chameleon field is rather heavy during inflation and has no influence on the effective mass of the inflaton field [21].

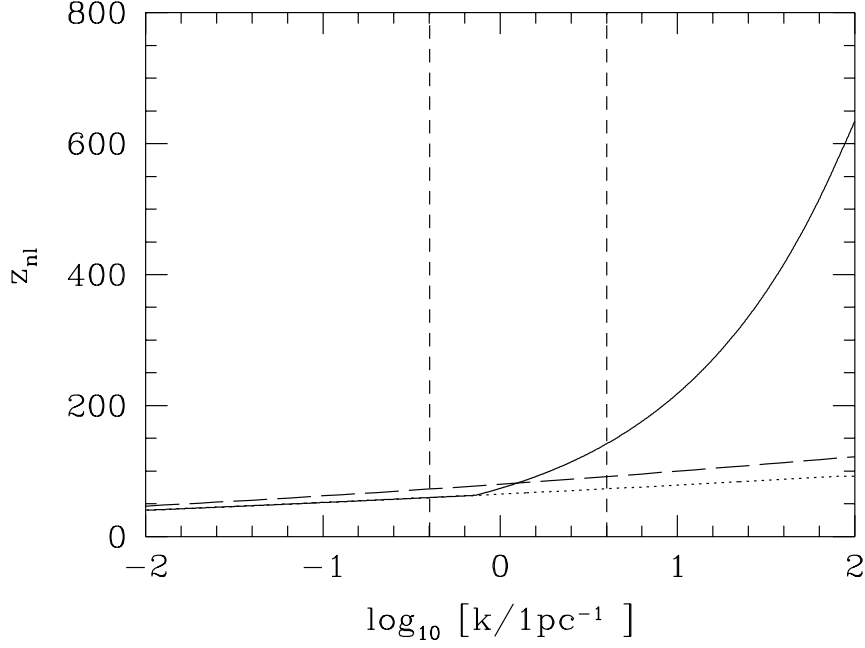


Figure 3: The red-shift at which typical $1\text{-}\sigma$ fluctuations collapse to form structure, $z_{\text{nl},\text{cham}}(k)$, for $\beta = 1$, $M = 10^{-3}\text{eV}$ and $n = 1$ (solid line). The dotted line is $z_{\text{nl},\text{GR}}(R = 1/k)$ and the short-dashed lines show the plausible range of k_{cut} values for WIMPs. The long-dashed line shows $z_{\text{nl},\text{GR}}(R = 1/k)$ for a primordial density perturbation power spectrum with spectral index $n_s = 1.036$.

earlier halos form the more over-dense they are at later times (reflecting the higher matter density at the time they form). In fig. 4 we plot the present day overdensity corresponding to typical halos, which form from $1\text{-}\sigma$ fluctuations, Δ ,

$$\Delta = \frac{2M(R)}{\frac{4\pi}{3}r(R)^3\rho_{\text{m}}(t_0)}, \quad (30)$$

as a function of comoving scale where $M(R) = 1.6 \times 10^7 M_{\odot} (\Omega_{\text{m}} h^2 / 0.14) (R/\text{pc})^3$ is the mean mass within a sphere of comoving radius R and $r(R) = 0.53R/(1 + z_{\text{nl}})$ is the physical, post collapse, radius of a halo which forms from a typical fluctuation with comoving size R at red-shift z_{nl} . For the chameleon we make the approximation $R \sim 1/k$. The first halos to form in chameleon cosmology are significantly more concentrated than in standard cosmology (provided $k_{\text{cut}} > \mathcal{O}(1\text{pc}^{-1})$ if the CDM is in the form of WIMPs) and are hence more likely to resist disruption by dynamical processes (such as tidal disruption and interactions with stars). We therefore expect that the present day dark matter distribution on small scales would be more clumped than in standard cosmology and this could be detectable via axion detectors or WIMP direct and indirect detection experiments (the

survival probability of the first dark matter halos even in standard cosmology is the subject of ongoing studies [8, 10, 22]). This alone would not provide a ‘smoking gun’ for the chameleon, as other modifications of general relativity could also lead to enhanced growth of small scale density perturbations and hence small scale structure. However, combined with a detection of modified gravity within the solar system [6] the present day density distribution could be used to probe modifications of gravity, such as those due to the chameleon.

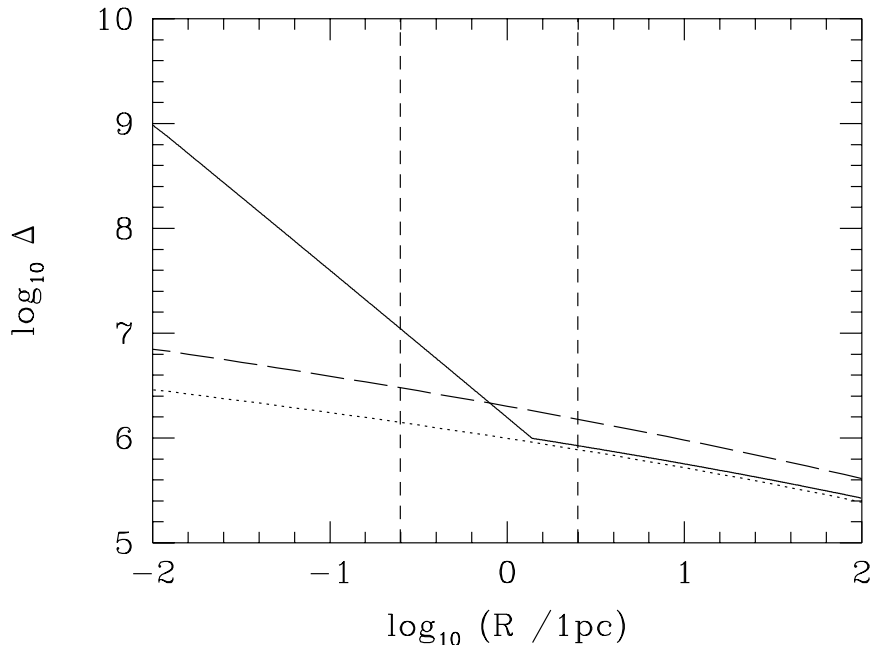


Figure 4: The present day overdensity corresponding to typical fluctuations, Δ , for the chameleon with $\beta = 1$, $M = 10^{-3}\text{eV}$ and $n = 1$ (solid line) and for standard cosmology with primordial power spectra which are scale independent (dotted) and $n_s = 1.036$ (long-dashed). The short-dashed lines show the range of $1/k_{\text{cut}}$ values for plausible WIMPs.

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